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WELCOME

Common Fixed Point Theorems for Single-valued and Multi-valued Maps

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Chapter 1 : Introduction and Preliminaries

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Chapter 2 : Common fixed point theorems for multi-valued and single-valued maps in complete metric space

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Chapter 1:Introduction and Preliminaries

Let X be a non-empty set and T be a self map on X. A point $x_0 \in X$ is called a fixed point of T if $Tx_0 = x_0$; that is, a point which remains invariant under the transformation T is called a fixed point of T.

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For example, let $T : [0, 1] \to [0, 1]$ be defined by $Tx = \frac{x}{10}$, Then $T(0) = 0$ and hence 0 is a fixed point of T

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Fixed point theorems serve as a powerful tool for taking these type of problems. It has also found diverse applications in areas like game theory, approximation theory, mathematical economics, Theory of differential equations etc.

Common fixed point theorems for multi-valued and single-valued maps in complete metric space

Let (X, d) **be a metric space. Denote by** $CB(X)$ **the collection of non-empty closed bounded subsets of** X**. For** $A, B \in CB(X)$ **and** $x \in X$**, define**

$$D(x, A) = \inf_{a \in A} d(x, a)$$

and

$$H(A, B) = max \left\{ \sup_{a \in A} D(a, B), \sup_{b \in B} D(b, A) \right\}$$

It is seen that H is a metric on CB(X). H is called the Hausdroff metric induced by d. It is well known that $(CB(X), H)$ is a complete metric space, whenever (X, d) is a complete metric space.

Definition

(see [1]) Let $T : X \to CB(X)$ be a multi-valued map. An element $x \in X$ is said to be fixed point of T if $x \in Tx$.

Definition

(see [1]) Maps $f : X \to X$ and $T : X \to CB(X)$ are weakly compatible if they commute at their coincidence points, that is, if $fTx = Tfx$ whenever $fx \in Tx$.

Definition

(see [1]) An element $x \in X$ is a common fixed point of $T, S : X \to CB(X)$ and $f : X \to X$ if $x = fx \in Tx \cap Sx$.

Definition

(see [2]) Definition 4: Let $\Phi : [0, \infty)^5 \to [0, \infty)$ be continuous (or upper semi-continuous)and increasing in each coordinate variable and $\Phi(t, t, t, at, bt) \leq$ t for every $t \in [0, \infty)$,where $a + b = 2,\ a, b \in \{0, 1, 2\}$

Theorem

Let (X, d) be a complete metric space and let $S, T : X \to CB(X)$ be a pair of multivalued maps and $f, g : X \to X$ be a pair of single valued maps. Assume that there exists $r \in [0, 1)$ such that for every $x, y \in X$,

$$\phi(r)min\{D(fx, Sx), D(gy, Ty)\} \leq d(fx, gy)$$

where

$$\phi(r) = \begin{cases} 1, & 0 \leq r < \frac{1}{2}, \\ (1 - r), & \frac{1}{2} \leq r < 1 \end{cases}$$

implies

$$H(Sx, Ty) \leq r\Phi\{d(fx, gy), D(fx, Sx), D(gy, Ty), D(fx, Ty), D(gy, Sx)\} \tag{1}$$

Suppose also that

(1) $SX \subseteq gX$, $TX \subseteq fX$

(2) $f(X)$ *and* $g(X)$ *are closed.*

Theorem

Then, there exists a point u and w in X, such that $fu = gw$, $fu \in Su$, $gw \in Tw$.(Here Φ is as specified in definition 4)

Theorem

Let (X, d) be a complete metric space and let $S, T : X \to CB(X)$ be a pair of multivalued maps and $f, g : X \to X$ be a pair of single valued maps. Assume that there exists $r \in [0, 1)$ such that for every $x, y \in X$,

$$H(Sx, Ty) \leq r\Phi\{d(fx, gy), D(fx, Sx), D(gy, Ty), D(fx, Ty), D(gy, Sx)\} \tag{2}$$

Suppose also that

(1) $SX \subseteq gX$, $TX \subseteq fX$

(2) $f(X)$ *and* $g(X)$ *are closed.*

Then, there exists a point u and w in X, such that $fu = gw$, $fu \in Su$, $gw \in Tw$ and $Su = Tw$(Here Φ is as specified in definition 4)

Theorem

Let (X, d) *be a complete metric space and let* $S, T : X \to CB(X)$ *be a pair of multivalued maps and* $f, g : X \to X$ *be a pair of single valued maps. Assume that there exists* $r \in [0, 1)$ *such that for every* $x, y \in X$,

$$\phi(r) min\{D(fx, Sx), D(gy, Ty)\} \leq d(fx, gy)$$

where

$$\phi(r) = \begin{cases} 1, & 0 \leq r < \frac{1}{2}, \\ (1-r), & \frac{1}{2} \leq r < 1 \end{cases}$$

implies

$$H(Sx, Ty) \leq r\Phi\{d(fx, gy), D(fx, Sx), D(gy, Ty), D(fx, Ty), D(gy, Sx)\} \tag{3}$$

Suppose also that

(1) $SX \subseteq gX$, $TX \subseteq fX$

(2) $f(X)$ *and* $g(X)$ *are closed.*

Theorem

Then, there exists a point u in X, such that $fu = gu$, $fu \in Su$, $gu \in Tu$. (Here Φ is as specified in definition 4)

Fixed point theorems on G-metric spaces and partial metric spaces

This Chapter focuses fixed point theorems in G-metric spaces and fixed point theorems in G-metric spaces by using the property P. Also this Chapter discusses fixed point theorems for generalized contractions on partial metric space.

Definition

(see [1]) Let X be a nonempty set, and let $G : X \times X \times X \to \mathbb{R}^+$ be a function satisfying the following axioms:

(G1) $G(x, y, z) = 0$ if $x = y = z$,

(G2) $0 < G(x, x, y)$ for all $x, y \in X$ with $x \neq y$,

(G3) $G(x, x, y) \leq G(x, y, z)$, for all $x, y, z \in X$, with $z \neq y$,

(G4) $G(x, y, z) = G(x, z, y) = G(y, z, x) = \cdots$ (symmetry in all three variables),

(G5) $G(x, y, z) \leq G(x, a, a) + G(a, y, z)$, for all $x, y, z, a \in X$ (rectangle inequality).

Then the function G is called a generalized metric, or, more specifically, a G-metric on X, and the pair (X, G) is called a G-metric space.

Definition

A partial metric on a nonempty set X is a function $p : X \times X \to \mathbb{R}_+$ such that for all $x, y, z \in X$:

(p1) $x = y \Longleftrightarrow p(x,x) = p(x,y) = p(y,y)$,

(p2) $p(x,x) \leq p(x,y)$,

(p3) $p(x,y) = p(y,x)$,

(p4) $p(x,y) \leq p(x,z) + p(z,y) - p(z,z)$

A partial metric space is a pair (X, p) such that X is a nonempty set and p is a partial metric on X.

Theorem

Theorem 14: Let (X, G) be a complete G-metric space, and let T be a self-map of X satisfying, for all $x, y, z \in X$,

$$G(Tx, Ty, Tz) \leq k \max\Big\{G(x,y,z), G(x,Tx,Tx), G(y,Ty,Ty), G(z,Tz,Tz), \frac{G(x,Ty,Ty)+G(z,Tx,Tx)}{2}, \frac{G(x,Ty,Ty)+G(y,Tx,Tx)}{2}, \frac{G(x,Tz,Tz)+G(z,Tx,Tx)}{2}\Big\} \quad (4)$$

where k is a constant satisfying $0 \leq k < 1$. Then T has a unique fixed point (say p) and T is G-continuous at p.

Theorem

Theorem 15: Let (X,G) be a complete G-metric space, and let T be a self-map of X satisfying, for all $x, y, z \in X$,

$$G(Tx, Ty, Tz) \leq k \max\Big\{G(x,y,z), G(x,Tx,Tx), G(y,Ty,Ty), G(x,Ty,Ty), G(z,Tz,Tz), \frac{G(x,Ty,Ty)+G(z,Tx,Tx)}{\alpha}, \frac{G(x,Ty,Ty)+G(y,Tx,Tx)}{\beta}, \frac{G(x,Tz,Tz)+G(z,Tx,Tx)}{\gamma}\Big\}, \tag{5}$$

where k is a constant satisfying $0 \leq k < 1$ and $S = \{(\alpha, \beta, \gamma \mid \alpha, \beta, \gamma \in (0,1]\}$, $\delta \in S$. Then T has a unique fixed point (say p) and T is G-continuous at p.

Theorem

Theorem 16: *Let (X, G) be a complete G-metric space, and let T be a self-map of X satisfying, for all $x, y, z \in X$,*

$$G(Tx, Ty, Tz) \leq k \max\Big\{G(x,y,z), G(x,Tx,Tx), G(y,Ty,Ty), G(z,Tz,Tz), \alpha[G(x,Ty,Ty)+G(z,Tx,Tx)], \alpha[G(x,Ty,Ty)+G(y,Tx,Tx)], \alpha[G(x,Tz,Tz)+G(z,Tx,Tx)]\Big\}, \qquad (6)$$

where k is a constant satisfying $0 \leq k < 1$ and $\alpha \in (0,1]$. Then T has a unique fixed point (say p) and T is G-continuous at p.

Theorem

Theorem 17: *Let (X,G) be a complete G-metric space, and let T be a self-map of X satisfying, for all $x, y, z \in X$,*

$$G(Tx, Ty, Tz) \leq k \max\Big\{G(x,y,z), \alpha[G(x, Tx, Tx) + G(y, Ty, Ty)], \alpha[G(x, Ty, Ty) + G(y, Tx, Tx)]\Big\}, \quad (7)$$

or

$$G(Tx, Ty, Tz) \leq k \max\Big\{G(x,y,z), \alpha[G(x,x, Tx) + G(y,y, Ty)], \alpha[G(x,x, Ty) + G(y,y, Tx)]\Big\}, \quad (8)$$

where k is a constant satisfying $0 \leq k < 1$ and $\alpha \in (0,1]$. Then T has a unique fixed point (call it p) and T is G-continuous at p.

Let T be a self-map of a complete metric space (X,d) with a nonempty fixed point set F(T). Then T is said to satisfy property P if $F(T) = F(T^n)$ for each $n \in \mathbb{N}$.

Theorem

Under the conditions of theorem 16, T has property P.

Theorem

Under the conditions of theorem 17, T has property P.

Theorem

Suppose A, B, S and T are self maps of a complete partial metric space (X, p) such that $BX \subseteq SX$, $AX \subseteq TX$ and

$$p(Bx, Ay) \leq \phi(M(x, y)) \tag{9}$$

for all $x, y \in X$, where $\phi \in \Phi$ and

$$M(x, y) = max\left\{p(Sy, Tx), p(Ay, Sy), p(Bx, Tx), \frac{1}{2}[p(Sy, Bx) + p(Ay, Tx)]\right\} \tag{10}$$

If one of the ranges AX, BX, SX and TX is a closed subset of (X, p), then

(p1) *B and T have a coincidence point,*

(p2) *A and S have a coincidence point.*

Moreover, if the pairs $\{B, T\}$and$\{A, S\}$ are weakly compatible, then A, B, S and T have a unique common fixed point.

Some common fixed point theorems for (ψ, φ)-weak contractive conditions on symmetric spaces for non-self mappings

This chapter deals with common fixed point theorems for non-self mappings (ψ, φ)-weak contractive conditions of integral type in symmetric spaces which is given by Kutbi.et.al[4]. A symmetric on a set X is a function $d : X \times X \to [0, \infty)$ satisfying the following conditions:
(1)$d(x, y) = 0$ **if and only if** $x = y$ **for** $x, y \in X$**,**
(2)$d(x, y) = d(y, x)$ **for all** $x, y \in X$**.**
From now on symmetric space will be denoted by (X, d) **whereas a non-empty arbitrary set will be denoted by** Y**.**
(W$_3$) Given $\{x_n\}$, x **and** y **in** X**,** $\lim_{n\to\infty} d(x_n, x) = 0$ **and** $\lim_{n\to\infty} d(x_n, y) = 0$ **imply x=y [2].**
(W$_4$) Given $\{x_n\}$**,** $\{y_n\}$ **and** x **in** X**,** $\lim_{n\to\infty} d(x_n, x) = 0$ **and** $\lim_{n\to\infty} d(x_n, y_n) = 0$ **imply** $d(y_n, x) = 0$ **[2].**
(HE) Given $\{x_n\}$**,** $\{y_n\}$ **and x in X,** $\lim_{n\to\infty} d(x_n, x) = 0$ **and** $\lim_{n\to\infty} d(y_n, x) = 0$ **imply** $d(x_n, y_n) = 0$ **[3].**

(1C) A symmetric d **is said to be** $1 - \textit{continuous}$ **if** $\lim_{n\to\infty} d(x_n, x) = 0$ **implies** $\lim_{n\to\infty} d(x_n, y) = d(x, y)$**, where** $\{x_n\}$ **is a sequence in X and** $x, y \in X$ **[4].**

(CC) A symmetric d is said to be continuous if $\lim_{n\to\infty} d(x_n, x) = 0$ **and** $\lim_{n\to\infty} d(y_n, y) = 0$ **imply** $\lim_{n\to\infty} d(x_n, y_n) = d(x, y)$ **where** $\{x_n\}$ **and** $\{y_n\}$ **are sequences in** X **and** $x, y \in X$ **[4].**

Definition

(see [4]) Let (A, S) be a pair of self-mappings defined on a non-empty set X equipped with a symmetric d.Then the mappings A and S are said to be
(1)commuting if ASx=SAx for all $x \in X$,
(2)compatible[5]if $\lim_{n\to\infty} d(ASx_n, SAx_n) = 0$ for each sequence $\{x_n\}$ in Y such that $\lim_{n\to\infty}$ Ax_n=$\lim_{n\to\infty}$ Sx_n
(3)non-compatible[1]if there exists a sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n$ but $\lim_{n\to\infty} d(ASx_n, SAx_n)$ is either non-zero or non-existent,
(4)weakly compatible[2] if they commute at their coincidence points, that is,
$ASx = SAx$ whenever $Ax = Sx$, for some $x \in X$,
(5)satisfying the property (E.A)[3] if there exists a sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = z$, for some $z \in X$.
Any pair of compatible as well as non-compatible self-mappings satisfies the property $(E.A)$ but a pair of mappings satisfying the property $(E.A)$ needs not be non-compatible.

Definition

(see [4]). Let Y be an arbitrary set and let X be a nonempty set equipped with symmetric d. Then the pairs (A, S) and (B, T) of mappings from Y into X are said to share the common property $(E.A)$, if there exist two sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = \lim_{n\to\infty} By_n = \lim_{n\to\infty} Ty_n = z \tag{11}$$

for some $z \in X$.

Definition

(see [3]). Let Y be an arbitrary set and let X be a non-empty set equipped with symmetric d. Then the pairs (A, S) of mappings from Y into X is said to have the common limit range property with respect to the mappings S (denoted by (CLR_S) if there exist two sequence $\{x_n\}$ in X such that

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = z \tag{12}$$

for some $z \in S(Y)$.

Definition

(see [4]). Let Y be an arbitrary set and let X be a non-empty set equipped with symmetric d. Then the pairs (A, S) and (B, T) of mappings from Y into X is said to have the common limit range property with respect to the mappings S and T, (denoted by (CLR_{ST})) if there exist two sequences $\{x_n\}$ and $\{y_n\}$ in Y such that

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = \lim_{n\to\infty} By_n = \lim_{n\to\infty} Ty_n = z \quad (13)$$

for some $z \in S(Y) \cap T(Y)$.

Lemma

Let (X, d)be a symmetric space wherein d satisfies the conditions (CC) whereas Y is an arbitrary nonempty set with A, B, S and $T : Y \rightarrow X$. Suppose that
(1)the pair $(A, S)(or(B, T))$ satisfies the $(CLR_S)(or(CLR_T))$ property,
(2)$A(Y) \subset T(Y)$,
(3)$T(Y)(orS(Y))$ is a closed subset of X,
(4)$\{By_n\}$ converges for every sequence $\{y_n\}$ in Y whenever $\{Ty_n\}$ converges (or $\{Ax_n\}$ converges for every sequence $\{x_n\}$ in Y whenever $\{Sx_n\}$ converges),
(5)there exists $\varphi \in \Phi$ and $\psi \in \Psi$ such that for all $x, y \in Y$, we have

$$\psi\left(\int_0^{d(Ax,By)} \phi(t)dt\right) \leq \psi\left(\int_0^{m(x,y)} \phi(t)dt\right) - \varphi\left(\int_0^{m(x,y)} \phi(t)dt\right)$$

where

$$m(x, y) = maxM^5_{A,B,S,T}(x, y)$$

Lemma

$$M^5_{A,B,S,T}(x,y) = [\{d^2(Sx,Ty), d^2(Sx,Ax), d(By,Ty), d(Sx,Ty), d(Sx,By), d(Sx,Ty), d(By,Ty), d^2(By,Ty)\}]^{\frac{1}{2}},$$

and $\phi : [0,\infty) \to [0,\infty)$ *is a lebesgue-integrable mapping which is summable and nonnegative such that*

$$\int_0^{\epsilon} \phi(t)dt > 0,$$

for all $\epsilon > 0$.
Then the pairs (A,S) *and* (B,T) *satisfy the* (CLR_{ST}) *property.*

Theorem

Let (X, d) be a symmetric space wherein d satisfies the conditions $(1C)$ and (HE) whereas Y is an arbitrary nonempty set with $A, B, S, T : Y \to X$, which satisfy the inequalities A and B of Lemma 1. Suppose that the pairs (A, S) and (B, T) satisfy the (CLR_{ST}) property. Then (A, S) and (B, T) have a coincidence point each. Moreover, if $Y = X$, then A, B, S and T have a unique common fixed point provided both the pairs (A, S) and (B, T) are weakly compatible.

Some common fixed point theorems for multivalued mappings under generalized contractive conditions

This chapter focuses fixed point theorems for multivalued mappings under generalized contractive conditions which generalize the results of Seong-Hoon Cho [5].

Let (X, d) be a metric space.We denote by $CB(X)$ the class of nonempty closed and bounded subsets of X and by CL(X) the class of nonempty closed subsets of X. Let $H(\cdot,\cdot)$ be the generalized Hausdorff distance on CL(X); that is, for all $A, B \in CL(X)$,

$$H(A,B) = \begin{cases} max\left\{\sup_{a\in A} d(a,B), \sup_{b\in B} d(b,A)\right\}, if \quad the \quad maximum \quad ex \\ \infty, \quad otherwise; \end{cases} \tag{14}$$

where $d(a, B) = inf\{d(a, b) : b \in B\}$ is the distance from point a to subset B. For $A, B \in CL(X)$, let $D(A, B) = sup_{x\in A} inf_{y\in B} d(x, y)$ Then,we have $D(A, B) \leq H(A, B)$ for all $A, B \in CL(X)$.
From now on, we denote by

$$M(x,y) = max\left\{d(x,y), d(x,Tx), d(y,Ty), \frac{d(x,Tx).d(y,Ty)}{1+d(x,y)}\right\} \tag{15}$$

for a multivalued map $T : X \to CL(X)$ **and** $x, y \in X$**.**
We denote by Ξ **the class of all functions** $\xi : [0, \infty) \to [0, \infty)$ **such that**
(1)ξ **is continuous;**
(2)ξ **is nondecreasing on** $[0, \infty)$**;**
(3)$\xi(t) = 0$ **if and only if** $t = 0$**;**
(4)ξ **is subadditive.**
Also,we denote by Ψ **the family of all nondecreasing functions** $\Psi : [0, \infty) \to [0, \infty)$ **such that** $\Sigma_{n=1}^{\infty} \psi^n(t) < \infty$ **for each** $t > 0$**, where** ψ^n **is the nth iterate of** ψ**.**
Note that if $\psi \in \Psi$**, then** $\mathit{\Psi}(0) = 0$ **and** $0 < \mathit{\Psi}(t) < t$ **for all** $t > 0$**.**

Let (X, d) be a metric space, and let $\alpha : X \times X \to [0, \infty)$ be a function.

We consider the following conditions:

(1)for any sequence $\{x_n\}$ in X with $\alpha(x_n, x_{n+1}) \geq 1$ for all $n \in \mathbb{N}$ and $lim_{n\to\infty} x_n = x$, we have

$$\alpha(x_n, x) \geq 1 \quad \forall n \in \mathbb{N} \tag{16}$$

(2)for any sequence $\{x_n\}$ in X with $\alpha(x_n, x_{n+1}) \geq 1$ for all $n \in \mathbb{N}$ and a cluster point x of $\{x_n\}$,we have

$$lim_{n\to\infty} inf\, \alpha(x_n, x) \geq 1; \tag{17}$$

(3)for any sequence $\{x_n\}$ in X with $\alpha(x_n, x_{n+1}) \geq 1$ for all $n \in \mathbb{N}$ and a cluster point x of $\{x_n\}$, there exists a subsequence $\{x_{n(k)}\}$ of $\{x_n\}$ such that

$$\alpha(x_{n(k)}, x) \geq 1 \quad \forall k \in \mathbb{N} \tag{18}$$

Note that if (X, d) is a metric space and $\xi \in \Xi$, then $(X, \xi \circ d)$ is a metric space.

Let (X, d) be a metric space, and let $T : X \to CL(X)$ be a multivalued mapping.Then,we say that

(1) T is called α_* -admissible [1] if

$$\alpha(x, y) \geq 1 \quad implies \qquad \alpha_*(Tx, Ty) \geq 1, \tag{19}$$

where $\alpha_*(Tx, Ty) = inf\ \{\alpha(a, b) : a \in Tx, b \in Ty\}$;

(2) T is called α-admissible [2] if, for each $x \in X$ and $y \in Tx$ with $\alpha(x, y) \geq 1$, we have $\alpha(y, z) \geq 1$ for all $z \in Ty$.

Theorem

Let (X, d) be a complete metric space, and let $\alpha : X \times X \to [0, \infty)$ be a function. Suppose that a multivalued mapping $T : X \to CL(X)$ is α-admissible.
Assume that ,for all $x, y \in X, \alpha(x, y) \geq 1$ implies

$$\xi(H(Tx, Ty)) \leq \psi(\xi(M(x, y))) + L\xi(d(y, Tx)) \tag{20}$$

$$M(x, y) = max\left\{d(x, y), d(x, Tx), d(y, Ty), \frac{d(x, Tx)d(y, Ty)}{1 + d(x, y)}\right\}$$

where $L \geq 0$,$\xi \in \Xi$,and $\psi \in \Psi$ is strictly increasing.
Also, suppose that the following are satisfied:
(1)there exists $x_0 \in X$ and $x_1 \in Tx_0$ such that $\alpha(x_0, x_1) \geq 1$;
(2)either T is continuous or f_T is lower semicontinuous.
Then T has a fixed point in X.

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Theorem

Let (X,d) *be a complete metric space and let* $\alpha : X \times X \to [0,\infty)$ *be a function. Suppose that a multivalued mapping* $T : X \to CL(X)$ *is* $\alpha -$ *admissible.*
Assume that, for all $x, y \in X$, $\alpha(x,y) \geq 1$ *implies*

$$\xi(H(Tx,Ty)) \leq \psi(\xi(M(x,y))) + L\xi(d(y,Tx)) \tag{21}$$

$$M(x,y) = max\left\{d(x,y), d(x,Tx), d(y,Ty), \frac{d(x,Tx)d(y,Ty)}{1+d(x,y)}\right\}$$

where $L \geq 0$,$\xi \in \Xi$,$\psi \in \Psi$ *is strictly increasing and upper semicontinuous function. Also, suppose that the following are satisfied:*
(1)there exists $x_0 \in X$ *and* $x_1 \in Tx_0$ *such that* $\alpha(x_0,x_1) \geq 1$;
(2)for a sequence $\{x_n\}$ *in* X *with* $\alpha(x_n,x_{n+1}) \geq 1 foralln \in \aleph \cup \{0\}$ *and a cluster point* x *of* $\{x_n\}$*,there exists a subsequence* $\{x_{n(k)}\}$ *of* $\{x_n\}$ *such that,for all ,* $k \in \mathbb{N} \cup \{0\}$,

Theorem

$$\alpha(x_{n(k)}, x) \geq 1. \tag{22}$$

Then T has a fixed point in X.

List of Publications

- **U . Karuppiah and A . Leema Maria Prakasam ,Fixed Point theorems in G-Metric Spaces,JP Journal of Fixed Point Theory and Applications , Vol. 8 , No.3 , pp. 165-191 .(2013)**
- **U . Karuppiah and A . Leema Maria Prakasam ,Common Fixed Point Theorems of Generalized Contractions on Partial Metric Spaces ,International Journal of Pure and Engineering Mathematics (IJPEM),Vol. 2, No . III , pp.59-70 .(2014)**
- **U . Karuppiah and A . Leema Maria Prakasam , Some Common Fixed Point Theorems for Non-Self Mappings (ψ, φ)-Weak Contractive Conditions of Integral Type in Symmetric Spaces , Far East Journal of Mathematical Sciences (FJMS),Vol. 98 , No . 1 , pp.15-35 .(2015)**
- **U . Karuppiah and A . Leema Maria Prakasam , New Common Fixed Point Theorems for Multi-valued and Single-valued Maps in Complete Metric Spaces ,International Journal of Pure and Engineering Mathematics (IJPEM), Vol. 3, No . I , pp. 263-281 .(2015)**

List of Publications

- **U . Karuppiah and A . Leema Maria Prakasam , Some Common Fixed Point Theorems for Multi-valued Maps in Complete Metric Spaces ,International Journal of Mathematical Sciences and Engineering Applications (IJMSEA), Vol. 9, No . II , pp. 157-178 .(2015)**
- **U . Karuppiah and A . Leema Maria Prakasam , Fixed Point Theorems for (α, ψ, ξ) Multi-valued Mappings ,International Journal of Mathematical Sciences and Engineering Applications (IJMSEA), Vol. 10 , No . II , pp. 19-30 .(2016)**

REFERENCES

Bosko Damjanovic,Bessem Samet,Calogero Vetro "Common fixed point theorems for multi-valued maps,"Science Direct Acta Mathematatica Scientia,vol.32B,no.2,pp.818-824,2012.

Ramendra Krishna Bose, "Some Suzuki type fixed point theorems for multivalued mappings and applications,"International Journal of Pure and Applied Mathematics,vol.92,no.4,pp.481-487,2014.

Ljubomir Ciric,Bessem Samet,Hassen Aydi,Calogero Vetro,Common Fixed Points of generalized contractions on partial metric spaces and an application,Applied Mathematics and computation 218(2011)2398-2406.

Marwan Amin Kutbi,Mohammad Imdad,Sunny Chauhan and Wutiphol Sintunavarat,"Some Integral Type Fixed Point Theorems for Non-Self- Mappings Satisfying Generalized (ψ, φ)-weak contractive conditions in Symmetric

Z. Mustafa and B. Sims, "A new approach to generalized metric spaces", Journal of Nonlinear and Convex Analysis, vol.7, no.2, pp.289-297, 2006.

W.A.Wilson,"On semi-metric spaces,"American Journal of Mathematics,vol.53,no.3,pp. 625-645,2012.

A.Aliouche,"A common fixed point theorem for weakly compatible mappings in symmetric spaces satisfying a contractive condition of integral type,"Journal of Mathematical Analysis and Applications,vol.322,no.2,pp.796-802,2006.

F.Galvin and S.D.Shore,"Completeness in semimetric spaces,"Pacific Journal of Mathematics and Mathematical Sciences,vol.113,no.1,pp.67-75,1984.

- H.Asl,S.Rezapour, and N.Shahzad"On fixed points of $\alpha - \psi$ contractive multifunctions ,"Fixed Point Theory and Applications,vol.2012,article 212,2012.
- B.Mohammadi,S.Rezapour,and N.shahzad,"Some results on fixed points of $\alpha - \psi$-Ciric generalized multifunctions,"Fixed Point Theory and Application,vol.2013,article 24,2013.
- W.Sintunavarat and P.Kumam,"Common fixed point theorems for a compatible mappings in fuzzy metric spaces,"Journal of Applied Mathematics,vol.2011,Article ID 637958,14 pages,2011.
- Marwan Amin Kutbi,Mohammad Imdad,Sunny Chauhan and Wutiphol Sintunavarat,"Some Integral Type Fixed Point Theorems for Non-Self- Mappings Satisfying Generalized (ψ, φ)-weak contractive conditions in Symmetric Spaces,"Hindawi Publishing Corporation,vol.2014,article ID 519038,2014.
- G.Jungck,"Compatible mappings and common fixed points,"International Journal of Mathematics and Mathematical

- **R.P.Pant,"Noncompatible mappings and common fixed points,"Soochow Journal of Mathematics,vol.26,no.1,pp.29-35,2000.**
- **G.Jungck and B.E.Rhoades ,"Fixed points for set valued functions without continuity,"Indian Journal of Pure and Applied Mathematics,vol.29,no.3,pp.227-238,1998.**
- **M.Aamri and D.El Moutawakil,"Some new common fixed pont theorems under strict contractive conditions,"Journal of Mathematical Analysis and Applications,vol.270,no.1,pp.181-188,2002.**
- **Y.Liu,J.Wu, and Z.Li,"Common fixed points of single-valued and multivaluedmaps,"International Journal of Mathematics and Mathematical Sciences,no.19,pp.3045-3055,2005.**

THANK YOU