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WELCOME

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OUTLINE OF THE THESIS

Chapter 1 : Introduction and Preliminaries

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Chapter 3 : Best proximity points in Multiplicative Metric Spaces and Multivalued mappings on Metric Spaces.

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Chapter 1:Introduction and Preliminaries

Fixed point theory is a beautiful mixture of analysis, topology and geometry. It has been a flourishing area of research for many mathematicians.

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Let X be a non-empty set and T be a self map on X. A point $x_0 \in X$ is called a fixed point of T if $Tx_0 = x_0$; that is, a point which remains invariant under the transformation T is called a fixed point of T.

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Let X be a non-empty set and T be a self map on X. A point $x_0 \in X$ is called a fixed point of T if $Tx_0 = x_0$; that is, a point which remains invariant under the transformation T is called a fixed point of T.

For example, let $T : [0, 1] \to [0, 1]$ be defined by $Tx = \frac{2x}{7}$. Then $T(0) = 0$ and hence 0 is a fixed point of T.

If T is not a self-mapping, it is plausible that the equation $x = Tx$ has no solution. In this situation, we may find an element $x \in X$ which is close to Tx in some sense.

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This point becomes a concept of best proximity point theorem.This theorem guarantees the existence of an element X such that

$$d(x, Tx) = inf\{d(a, b) : a \in A, b \in B\} = d(A, B),$$
$$A_0 = \{a \in A : d(a, b) = d(A, B) \text{ for some } b \in B\},$$
$$B_0 = \{b \in B : d(a, b) = d(A, B) \text{ for some } a \in A\},$$

then x is called a best proximity point of non-self mapping T.

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then x is called a best proximity point of non-self mapping T.

If $d(A, B) = 0$,then a fixed point and best proximity point are same point.If the mapping under consideration is a self mapping,it may be observed that a best proximity point theorem boils down to the fixed point theorem under certain suitable conditions.

Some best proximity theorems for $\alpha - \psi$ rational proximal contractive conditions in Multiplicative Metric Spaces.

Definition

Let X be a non-empty set. Multiplicative metric is a mapping $d : X \times X \to \mathbb{R}^+$ satisfying the following conditions such that for all $x, y, z \in X$:

(p1) $d(x, y) \geq 1$ and $d(x, y) = 1$ iff $x = y$,

(p2) $d(x, y) = d(y, x)$,

(p3) $d(x, z) \leq d(x, y).d(y, z)$

Then the function d is said to be a Multiplicative Metric on X and (X,d) is called a Multiplicative Metric Space.

Definition

Let (X, d) be a multiplicative metric space and A, B be two non-empty subsets of X. Let $T : A \to B$ and $\alpha : A \times A \to [0, \infty)$ be the functions. Then T is said to be $(\alpha - \psi)$ rational proximal contraction, if for all $x, y, u, v \in A$ and $\psi \in \Psi_3$ such that

$$\begin{aligned} d(u, Tx) &= d(A, B) \\ d(v, Ty) &= d(A, B) \implies \alpha(x, y) d(u, v) \leq \psi(M(x, y)) \end{aligned}$$

where,

Definition

$$M(x,y) = max\{d(x,y), \frac{d(x,Tx).d(y,Ty)}{1+d(x,y)} - d(A,B), \frac{d(x,Ty).d(y,Tx)}{1+d(x,y)} - d(A,B), \frac{d(x,Ty).d(y,Tx)}{1+d(Tx,Ty)} - d(A,B)\}$$

Theorem

Let A and B be a two non-empty closed subsets of a complete multiplicative metric space (X, d) such that A_0 and B_0 are non-empty. Let the mappings $\alpha : A \times A \to [0, \infty)$, $T : A \to B$ and $G : A \to A$ satisfy the following conditions.

- (i) T is $(\alpha - \psi)$ rational proximal contraction mapping and T is an α-proximal admissible mapping.
- (ii) g is an isometry
- (iii) $A_0 \subseteq g(A_0)$
- (iv) $T(A_0) \subseteq B_0$.
- (v) If $\{x_n\}$ is a sequence in A_0 such that $\alpha(gx_n, x_{n+1}) \geq 1$ and $gx_n \to gx \in A$, then $\alpha(x_n, gx) \geq 1$ for all $n \in \mathbb{N}$
- (vi) There exists x_0, $x_1 \in A_0$ such that $d(gx_1, Tx_0) = d(A, B)$ and $\alpha(x_0, x_1) \geq 1$

Then T has a unique best proximity point if for every $y, z \in A$ such that $d(gy, Ty) = d(A, B) = d(gz, Tz)$ and $\alpha(gy, gz) \geq 1$.

Corollary

Let A and B be a two nonempty closed subsets of a complete multiplicative metric space (X, d) such that A_0 and B_0 are nonempty. Let the mappings $\alpha, \eta : A \times A \to [0, \infty)$, $T : A \to B$ and $G : A \to A$ satisfy the following conditions.

- *(i)* *T is $(\alpha - \psi)$ rational proximal contraction mapping with respect to η.*
- *(ii)* *g is an isometry*
- *(iii)* $A_0 \subseteq g(A_0)$
- *(iv)* $T(A_0) \subseteq B_0$.
- *(v)* *If $\{x_n\}$ is a sequence in A_0 such that $\alpha(gx_n, x_{n+1}) \geq \eta(gx_n, x_{n+1})$ and $gx_n \to gx \in A$, then $\eta(x_n, gx) \geq \eta(x_n, gx)$ for all $n \in \mathbb{N}$*

There exists x_0, $x_1 \in A_0$ such that $d(gx_1, Tx_0) = d(A, B)$ and $\alpha(x_0, x_1) \geq \eta(x_0, x_1)$. Then T has a unique best proximity point if for every $y, z \in A$ such that $d(gy, Ty) = d(A, B) = d(gz, Tz)$ and $\alpha(gy, gz) \geq \eta(gy, gz)$.

Corollary

Let A and B be a two nonempty closed subsets of a complete multiplicative metric space (X, d) such that A_0 and B_0 are nonempty. Let the mappings $\eta : A \times A \to [0, \infty)$, $T : A \to B$ and $G : A \to A$ satisfy the following conditions.

(i) *T is ψ rational proximal contraction.*

(ii) *g is an isometry*

(iii) $A_0 \subseteq g(A_0)$

(iv) *$T(A_0) \subseteq B_0$ and T is η-subadmissible.*

(v) *If $\{x_n\}$ is a sequence in A_0 such that $\eta(gx_n, x_{n+1}) \leq 1$ and $gx_n \to gx \in A$, then $\eta(x_n, gx) \leq 1$ for all $n \in \mathbb{N}$*

There exists x_0, $x_1 \in A_0$ such that $d(gx_1, Tx_0) = d(A, B)$ and $\eta(x_0, x_1) \leq 1$ Then T has a unique best proximity point if for every $y, z \in A$ such that $d(gy, Ty) = d(A, B) = d(gz, Tz)$ and $\eta(gy, gz) \leq 1$.

Corollary

Let A and B be a two nonempty closed subsets of a complete multiplicative metric space (X, d) such that A_0 and B_0 are nonempty. Let the mappings $\alpha, \eta : A \times A \to [0, \infty)$, $T : A \to B$ and $G : A \to A$ satisfy the following conditions.

(i) *T is $(\alpha - \psi)$ rational proximal contraction mapping with respect to η.*

(ii) $T(A_0) \subseteq B_0$.

(iii) *If $\{x_n\}$ is a sequence in A_0 such that $\alpha(x_n, x_{n+1}) \geq \eta(x_n, x_{n+1})$ and $gx_n \to gx \in A$, then $\alpha(x_n, x_{n+1}) \geq \eta(x_n, x_{n+1})$ for all $n \in \mathbb{N}$*

(iv) *There exists x_0, $x_1 \in A_0$ such that $d(x_1, x_0) = d(A, B)$ and $\alpha(x_0, x_1) \geq \eta(x_0, x_1)$*

Then T has a unique best proximity point if for every $y, z \in A$ such that $d(y, Ty) = d(A, B) = d(z, Tz)$ and $\alpha(y, z) \geq \eta(y, z)$.

Theorem

Let T be a complete multiplicative metric space (X, d) into itself and $\alpha : A \times A \to [0, \infty)$ be a given function satisfying the following conditions.

- (i) *T is α-admissible mapping.*
- (ii) *T is continuous*
- (iii) *g is an isometry and $A \subseteq g(A)$ such that $\alpha(x, y) \geq 1$ and $d(Tx, Ty) \leq \psi(M(x, y))$*

where

Theorem

$$M(x,y) = max\{d(x,y), \frac{d(x,Tx).d(y,Ty)}{1+d(x,y)}, \frac{d(x,Ty).d(y,Tx)}{1+d(x,y)},$$
$$\frac{d(x,Ty).d(y,Tx)}{1+d(Tx,Ty)}\}$$

Then T has a unique fixed point.

Theorem

Let T be a complete multiplicative metric space (X,d) into itself and $\alpha : A \times A \to [0,\infty)$ be a given function satisfying the following conditions.

(i) *T is α-admissible mapping.*

(ii) *T is continuous*

(iii) *g is an isometry and $A \subseteq g(A)$ such that $\alpha(x,y) \geq 1$ and $d(Tx,Ty) \leq k(M(x,y))$*

where

Theorem

$$M(x,y) = max\{d(x,y), \frac{d(x,Tx).d(y,Ty)}{1+d(x,y)}, \frac{d(x,Ty).d(y,Tx)}{1+d(x,y)}, \frac{d(x,Ty).d(y,Tx)}{1+d(Tx,Ty)}\}$$

Then T has a unique fixed point.

Theorem

Let T be a complete multiplicative metric space (X, d) into itself and $\alpha : A \times A \to [0, \infty)$ be a given function satisfying the following conditions.

(i) T *is* α*-admissible mapping.*

(ii) T *is continuous*

(iii) g *is an isometry and* $A \subseteq g(A)$ *such that* $\alpha(x, y) \geq 1$ *and* $d(Tx, Ty) \leq kd(x, y)$

Then T *has a unique fixed point.*

Best proximity points in Multiplicative Metric Spaces and Multivalued mappings on Metric Spaces.

In this chapter, we focus best proximity point theorems in multiplicative metric spaces satisfying multiplicative modified rational proximal contraction condition of the first kind and also prove best proximity points for multivalued Geometric F - contraction mappings .

Definition

Let (X,d) be a multiplicative metric space. Let A and B be a nonempty subsets of X. Then $T : A \to B$ is called a multiplicative modified rational proximal contraction of the first kind if there exists a non-negative real numbers $\alpha, \beta, \gamma, \delta$ with $\alpha + \beta + 2\gamma + 2\delta < 1$ such that the conditions

$$d(u_1, Tx_1) = d(A, B) \text{ and } d(u_2, Tx_2) = d(A, B)$$

This implies

$$d(u_1, u_2) \leq \frac{d(x_1, x_2)^{\alpha}.[d(x_1, u_1).d(x_2, u_2)]^{\beta+\gamma}.[d(x_1, u_2).d(x_2, u_1)]^{\delta}}{d(x_1, x_2)}$$

for all $u_1, u_2, x_1, x_2 \in A$

Theorem

Let (X, d) be a complete multiplicative metric space. Let A, B be a nonempty closed subsets of X such that A_0 and B_0 are nonempty and B is approximately compact with respect to A. Suppose that $T : A \to B$ and $g : A \to A$ satisfy the following conditions:

a) *T is a multiplicative modified rational proximal contraction of the first kind*
b) $T(A_0) \subseteq B_0$
c) *g is an isometry*
d) $A_0 \subseteq g(A_0)$

Then there exists a unique point $x \in A$ such that

$$d(gx, Tx) = d(A, B)$$

Moreover for any fixed $x_0 \in A_0$, the sequence $\{x_n\}$ is defined by $d(gx_n, Tx_{n-1}) = d(A, B)$ converges to the element x.

Corollary

Let (X, d) be a complete multiplicative metric space. Let A, B be a nonempty closed subsets of X such that A_0 and B_0 are nonempty and B is approximately compact with respect to A. Suppose that $T : A \to B$ satisfy the following conditions:

a) *T is a multiplicative modified rational proximal contraction of the first kind*

b) $T(A_0) \subseteq B_0$

Then there exists a unique point $x \in A$ such that

$$d(x, Tx) = d(A, B)$$

Moreover for any fixed $x_0 \in A_0$, the sequence $\{x_n\}$ is defined by $d(x_n, Tx_{n-1}) = d(A, B)$ converges to the element x.

Definition

Let (X, d) be a metric space. Let $C_b(X)$ be the family of all non-empty closed bounded subsets of a metric space (X, d). The Hausdorff metric induced by d on $C_b(X)$ is given by

$$H(A, B) = max \left\{ \sup_{a \in A} d(a, B), \sup_{b \in B} d(b, A) \right\}$$

for every A,B $\in C_b(X)$,
where $d(a, B)$=inf$\{d(a, b) : b \in B\}$ is the distance from a to B$\subseteq$ X.

Definition

Let A and B be nonempty subsets of a metric space (X, d). The ordered pair (A, B) satisfies the property UC^{**} if (A, B) has property UC and the following conditions holds: If $\{x_n\}$ and $\{z_n\}$ are sequences in A and $\{y_n\}$ be a sequence in B satisfying

1. $d(z_n, y_n) \to d(A, B)$ as $n \to \infty$
2. For each $\epsilon > 0$, there exists $N \in \mathbb{N}$ such that
$$d(x_m, y_n) \leq \varepsilon d(A, B)$$
for all $m > n \geq N$

then $d(x_n, z_n) \to 0$ as $n \to \infty$.

Definition

Let A and B be non-empty subsets of a metric space.
Let $T : A \to 2^B$ and $S : B \to 2^A$ be multivalued mappings. The ordered pair (T, S) is said to be a multivalued Geometric F - contraction if there exists $F \in \mathbb{F}$ and $\tau > 0$ such that
$H(Tx, Sy) > 0 \Longrightarrow 2\tau + F(H(Tx, Sy)) \leq F(d(x, y)^{\alpha} dist(A, B)^{1-\alpha})$,
for all $x, y \in X$, where $\alpha \in (0, 1)$.

Theorem

Let A and B be non-empty closed subsets of a complete metric space X such that (A, B) and (B, A) satisfy the property UC^{**}.
Let $T : A \to C_b(B)$ and $S : B \to C_b(A)$. If (T, S) is a multivalued geometric F-contraction pair, then T has a best proximity point in A (or) S has a best proximity point in B.

Fixed point theorems in Partial Metric Spaces and Quasi Partial Metric Spaces.

Chapter 4

In this chapter, we prove some common fixed point theorems in partial metric spaces and quasi partial metric spaces.

Definition

A partial metric on a nonempty set X is a mapping $p : X \times X \to [0, \infty)$ such that for all $x, y, z \in X$:

(p1) $x = y \iff p(x,x) = p(x,y) = p(y,y)$,

(p2) $p(x,x) \leq p(x,y)$,

(p3) $p(x,y) = p(y,x)$,

(p4) $p(x,y) \leq p(x,z) + p(z,y) - p(z,z)$

A partial metric space is a pair (X, p) such that X is a nonempty set and p is a partial metric on X.

Definition

(see [1]) Two self mappings f and g of a set X are said to be weakly compatible if they commute at their coincidence points, that is, if $fx = gx$ for some $x \in X$,then $fgx = gfx$.

Definition

(see [1]) Let $(X,p,\preccurlyeq)$ be a partially ordered set. Two elements x,y of X are called comparable if $x \preccurlyeq y$ (or) $y \preccurlyeq x$ holds.

Definition

(see [1]) Let $(X,p,\preccurlyeq)$ be a partially ordered set.A mappings f is called weak annihilator of g if $fgx \preccurlyeq x$ for all $x \in X$

Definition

Let f,g,S and T be self maps on a partial metric space (X, p),then f and g are said to satisfy almost generalized (S,T)-contractive condition if there exists $\delta \in (0, 1)$ such that

$$p(fx, gy) \leq \delta M(x, y)$$

for all $x, y \in X$,where
$M(x, y) = max\{p(Sx, Ty), p(fx, Sx), p(gy, Ty), \alpha(p(Sx, gy) + p(fx, Ty))\}$,
$\alpha \in$(0,1).

Definition

Let f,g,S and T be self maps on a partial metric space (X,p),then f and g are said to satisfy almost generalized (S,T)-contractive condition if there exists $\delta \in (0,1)$ such that

$$p(fx, gy) \leq \delta M(x, y)$$

for all x,y∈X,where

M(x,y)=max{p(Sx,Ty),p(gx,Sx),p(gy,Ty),$\dfrac{p(Sx, Ty) + p(fx, Sy)}{2}$}

Definition

Let f,g,S and T be self maps on a partial metric space (X,p),then f and g are said to satisfy almost generalized (S,T)-contractive condition if there exists $\delta \in (0,1)$ such that

$$p(fx, gy) \leq \delta M(x, y)$$

for all x,y $\in$ X,where
$M(x, y) =$
$\psi\{max\{p(Sx, Ty), p(fx, Sx), p(gy, Ty), \frac{p(Sx, gy) + p(fx, Ty)}{2}\}\}$,
where $\psi : R^+ \to (0, 1)$ and satisfies $0 \leq \psi(t) \leq t$ for $t > 0$.

Theorem

Let $(X,p,\preccurlyeq)$ be an complete ordered partial metric space.Let f,g,S and T be self maps on X,with $f(X)\subseteq T(X)$ and $g(X)\subseteq S(X)$ and dominating maps f and g are weakly annihilators of T and S respectively.
Suppose that f and g satisfy almost generalised (S,T)- contractive condition

$$p(fx,gy)\leq \delta M(x,y)$$

for every two comparable elements $x,y \in X$.
If for a non-decreasing sequence $\{x_n\}$ with $x_n \preccurlyeq y_n$ for all n and $y_n \rightarrow u$ implies that $x_n \preccurlyeq u$ and further more.

(a1) $\{f,S\}$and$\{g,T\}$ *are weakly compatible,*

(a2) *one of* $f(x), g(x), S(x)$and$T(x)$ *is a closed subspace of X*

then f,g,S and T have a common fixed point. Moreover, the set of common fixed points of f,g,S,T is well ordered iff f,g,S and T have one and only one common fixed point.

Corollary

Let $(X,p,\preccurlyeq)$ *be an complete ordered partial metric space.Let f and T be self maps on X,with* $f(X)\subseteq T(X)$ *and dominating map f is weakly annihilators of T . Suppose that there exists* $\delta \in (0,1)$ *such that*

$$p(fx,fy)\leq \delta M(x,y)$$

where
$M(x,y) = \max\{p(Tx,Ty),p(fx,Tx),p(fy,Ty),\alpha(p(Tx,fy)+p(fx,Ty))\}$
for every two comparable elements $x,y \in X$*. If for a non-decreasing sequence* $\{x_n\}$ *with* $x_n \preccurlyeq y_n$ *for all n and* $y_n \to u$ *implies that* $x_n \preccurlyeq u$ *and further more.*

(a1) $\{f, T\}$ *is weakly compatible.*

(a2) *one of f(x) and T(x) is a closed subspace of X.*

then f and T have a common fixed point.

Corollary

Let $(X,p,\preccurlyeq)$ be an complete ordered partial metric space.Let S and T be surjective self maps on X,such that $S(x)\preccurlyeq x$ and $T(x)\preccurlyeq x$ for all $x\in X$, and Suppose that there exists $\delta \in (0,1)$ such that

$$p(x,y)\leq \delta M(x,y)$$

where
$M(x,y) = \max\{p(Sx,Ty),p(x,Sx),p(y,Ty),\alpha(p(Sx,y)+p(x,Ty))\}$
for every two comparable elements $x,y \in X$. If for a non-decreasing sequence $\{x_n\}$ with $x_n \preccurlyeq y_n$ for all n and $y_n \to u$ implies that $x_n \preccurlyeq u$,then S and T have a common fixed point.

Definition

A quasi-partial metric space on a nonempty set X is a function $q : X \times X \to [0, \infty)$ such that for all $x, y, z \in X$:

(p1) If $q(x,x) = q(x,y) = q(y,y)$ then $x = y$,

(p2) $q(x,x) \leq q(x,y)$,

(p3) $q(x,x)= q(y,x)$,

(p4) $q(x,z) \leq q(x,y) + q(y,z) - q(y,y)$

Definition

Let T be a self mapping and $\alpha : X \times X \to [0, +\infty)$ be a function. Then T is said to be α - Orbital admissible if

$$\alpha(x, Tx) \geq 1 \implies \alpha(Tx, T^2x) \geq 1$$

Definition

Let (X, q) be a quasi-partial metric space.where X is a non-empty set. we say that X is said to be α-left-regular if for every sequence $\{x_n\}$ in X such that $\alpha(x_{n+1}, x_n) \geq 1$ for all n and $x_n \to x \in X$ as $n \to \infty$, there exists a subsequence $\{x_{n(k)}\}$ of $\{x_n\}$ such that $\alpha(x, x_{n(k)}) \geq 1$ for all k.

Definition

Analogously, a quasi-partial metric space X is said to be an $\alpha-$right-regular if for every sequence $\{x_n\}$ in X such that $\alpha(x_n, x_{n+1}) \geq 1$ for all n and $x_n \to x \in X$ as $n \to \infty$, there exists a subsequence $\{x_{n(k)}\}$ of $\{x_n\}$ such that $\alpha(x_{n(k)}, x) \geq 1$ for all k.

Definition

We say that X is regular if it is both α-left-regular and α-right-regular.

Theorem

Let (X,q) be a complete quasi partial metric space.
Let $T : X \to X$ be a self-mapping. Assuming that there exists $\psi \in \Psi$ and a function $\alpha : X \times X \to [0, \infty)$ such that for all $x,y \in X$

$$\alpha(x, y)q(Tx, Ty) \leq \psi(M(x, y))$$

Also suppose that the following assertions hold:
(i) T is triangular α-orbital admissible.
(ii) there exists $x_0 \in X$ such that $\alpha(x_0, Tx_0) \geq 1$ and $\alpha(Tx_0, x_0) \geq 1$
(iii) T is continuous (or) X is α-regular.
Then T has a fixed point $u \in X$ and q(u,u)=0.

Some fixed point theorems on Multiplicative Cone-b metric spaces and Multi-valued mappings on b- metric spaces

Chapter 5

In this Chapter, we prove some fixed theorems using some Multiplicative contractive conditions in multiplicative cone b - metric spaces and also we prove a common fixed point theorem for multivalued mappings in b-metric spaces which is a generalization of a Reich type contraction .

Definition

Let X be a nonempty set and $s \geq 1$ be a given positive real number. A mapping $d : X \times X \to E$ such that for all $x, y, z \in X$:

(p1) $d(x, y) \geq 1$ and $d(x, y) = 1$ iff $x = y$,

(p2) $d(x, y) = d(y, x)$,

(p3) $d(x, y) \leq [d(x, z).d(z, y)]^s$

Then the function d is said to be a multiplicative cone b-metric on X and (X,d) is called a multiplicative cone b-metric space.

Theorem

Let (X, d) be a Complete multiplicative cone b - metric space with power $s \geq 1$. Suppose the mapping $T : X \to X$ satisfies the following Kannan contractive condition,

$$d(Tx, Ty) \leq (d(Tx, x).d(Ty, y))^{\lambda} \text{ for all } x, y \in X$$

where $0 \leq \lambda < \frac{1}{2}$ is a constant. Then T has a unique fixed point in X and for any $x \in X$, iterative sequence $\{T^n x\}$ converges to the fixed point.

Theorem

Let (X, d) be a Complete multiplicative cone b - metric space with power $s \geq 1$. Suppose the mapping $T : X \to X$ satisfies the contractive condition,

$$d(Tx, Ty) \leq (d(Tx, y).d(Ty, x))^{\lambda} \text{ for all } x, y \in X$$

where $0 \leq \lambda < \frac{1}{2}$ is a constant. Then T has a unique fixed point in X and for any $x \in X$, iterative sequence $\{T^n x\}$ converges to the fixed point.

Theorem

Let (X, d) be a complete cone b- metric space with metric d and $T : X \to X$ be a function with the following condition,

$$d(Tx, Ty) \leq d(x, Tx)^p.d(y, Ty)^q.d(x, y)^r,$$

for all $x, y \in X$, where p, q, r are non-negative real numbers and satisfy $p + (q + r)^s < 1$ for $s \geq 1$. Then T has a unique fixed point.

Definition

Let X be a nonempty set and $s \geq 1$ be a given positive real number.A mapping $d : X \times X \to \mathbb{R}$ such that for all $x, y, z \in X$:

(p1) $d(x, y) \geq 0$ and $d(x, y) = 0$ iff $x = y$,

(p2) $d(x, y) = d(y, x)$,

(p3) $d(x, y) \leq s[d(x, z) + d(z, y)]$

Then the function d is said to be a b-metric on X and (X,d) is called a b-metric space.

Definition

Let (X, d) be a b-metric space with constant $s \geq 1$. A map $T : X \to CB(X)$ is said to be multivalued generalized contraction if

$$H(Tx, Ty) \leq ad(x, y) + bd(x, Ty) + c(d(y, Tx) + d(Tx, Ty))$$

for all $x, y \in X$ and a, b, c are non-negative with $a + b + 2c < 1$.

Theorem

Let (X, d) be a complete b-metric space with constant $s \geq 1$. Let $T : X \to CB(X)$ is said to be multivalued generalized contraction mapping. Then T has a unique fixed point.

Theorem

Let (X, d) be a complete b-metric space with constant $s \geq 1$. Let $T : X \to CB(X)$ be a multivalued mapping satisfied the condition.

$$H(Tx, Sy) \leq ad(x, y) + bd(x, Sy) + c(d(y, Tx) + d(Tx, Ty))$$

for all $x, y \in X$ and a, b, c are non-negative with $a + b + 2c < 1$. Then T & S have a unique common fixed point.

List of Publications

1 . U.Karuppiah and A.Mary Priya Dharsini, Some fixed point theorems satisfying (S, T) contractive condition in partially ordered partial metric space, Far East Journal of Mathematical Sciences(FJMS), Volume 95, No.1, 2014, pp 19-50.ISSN 0972-0871.

2 . U.Karuppiah and A.Mary Priya Dharsini, Some theorems on $\alpha - \psi$ quasi contractive on quasi partial metric space, International Journal of Mathematical Archive-7(10), 2016, 185-191.ISSN 2229-5046.

3 . U.Karuppiah and A.Mary Priya Dharsini, On best proximity points for multivalued geometric F - contraction mappings, International Journal of Scientific and Engineering research, Vol: 8, Issue 7, July 2017.

4 . U.Karuppiah and A.Mary Priya Dharsini, Fixed Point theorems for multiplicative Contraction mappings on multiplicative Cone-b metric spaces, International journal of mathematics trends and technology, Volume 8, Part 2,Number 3,August 2017, PP 8-12. ISSN 2231-5373.

5 . U.Karuppiah and A.Mary Priya Dharsini, Best proximity points for multiplicative modified rational proximal contraction mapping on multiplicative metric spaces, International Journal of Mathematics and its applications.[Accepted]

6 . U.Karuppiah and A.Mary Priya Dharsini, Best proximity points theorems for $(\alpha - \psi)$ rational proximal contractions mappings in multiplicative metric spaces,International Journal of Engineering ,Science and Mathematics(IJESM),Vol-6,Issue 7,Nov -2017.ISSN 2320-0294.

7 . U.Karuppiah and A.Mary Priya Dharsini, Common Fixed Point Theorem for Multivalued Mappings on *b*-metric spaces, Journal of Global Research Mathematical Archieves[Accepted]

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THANK YOU